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Strictly flat cyclic Fréchet modules and approximate identities

Let A be a Banach algebra, A_+ its unitization, and $I \subset A_+$ a closed left ideal. Helemskii's theorem states that the cyclic Banach A-module $X = A_+/I$ is strictly flat if and only if I has a right bounded approximate identity. This theorem plays an important rôle in questions related to amenability. Our aim is to generalize this criterion to the case where A is a locally m-convex Fréchet algebra. To this end, we introduce a notion of locally bounded approximate identity (a.i.), and we show that X is strictly flat if and only if I has a right locally bounded a.i. We also give an example of a commutative locally m-convex Fréchet algebra that has a locally bounded a.i., but does not have a bounded a.i. On the other hand, we show that a distinguished locally m-convex Fréchet algebra has a locally bounded a.i. if and only if it has a bounded a.i. Some applications to amenable Fréchet algebras are also given.

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