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Frechet algebras with spectral invariance in the microlocal analysis

Some perturbation theory of holomorphic Fredholm functions with values in Frechet algebras of pseudodifferential operators is discussed.

A denotes a Frechet subalgebra of the Banach algebra $\mathcal{L}(E)$ for a Banach space E over $\mathbb{C}$ ($e = Id_E \in A$) such that the inverse of $a \in A$ with $||a - e||_{\mathcal{L}(E)} < 1$ is an element of A. (A is a 0-algebra). Let Ω be a holomorphy region in a DFN-space with basis and $T: \Omega \to \Phi^l(A)$ a holomorphic map with values in the set of Semi-Fredholm operators of A.

Theorem. If $T(z^*)$ is left invertible in A for some $z \in \Omega$. Then there exists a meromorphic left inverse M of T on with a global decomposition $M(z) = H(z) + S(z)$, $z \in \Omega$, where $H: \Omega \to A$ is holomorphic and S meromorphic with values in the ideal S(A) of operators $a \in \Omega$ with fast decreasing approximation numbers.

Remarks. 1) This applies to the L^p-theory of pseudodifferential operators.

2) The theorem above seems to be new for A a C*-algebra and $\dim \Omega = 1$.

3) A multiplicative analogue can be derived using the Oka principle in the non abelian cohomology of holomorphic cocycles.

4) The ideal S(A) can be substituted by much smaller ideals with some exponential decay.

A general method is described to produce submultiplicative Ψ_0-algebras with commutator methods starting from spaces of closed operators on Banach spaces respectively on Banach modules over rings of $C^?$ -functions.