Edward Kissin (Joint work with V. Shulman)

Lipschitz functions on Hermitian Banach *-algebras.

We characterize semisimple Hermitian Banach *-algebras A with "rich" spaces of A-Lipschitz functions and describe these spaces. A continuous function g on a compact α in $\mathbb{R}$ is an A-Lipschitz function, if there exists $D > 0$ such that

$$g(a),\ g(b) \in A \text{ and } \|g(a) - g(b)\|_A \leq D\|a - b\|_A,$$

for all selfadjoint $a,b \in A$ with spectra in α: If $A = B(H)$; then g is called an Operator Lipschitz function. Operator Lipschitz functions are differentiable at each point but not necessarily continuously differentiable.
For a unital Hermitian Banach *-algebra A; the presence of a non-linear A-Lipschitz function implies that A is a C^*-algebra. If A only has fnite-dimensional irreducible representations and their dimensions are bounded, then the space of A-Lipschitz functions coincides with the space of all Lipschitz in the usual sense functions. Otherwise, it coincides with the space of all Operator Lipschitz functions.
The non-unital case is more varied. There is a large class of non-C^*-equivalent algebras whose spaces of A-Lipschitz functions contain all functions with Fourier transform $\hat{g}$ satisfying $\int |t\hat{g}(t)|\,dt < \infty$. They are defined by a very mild condition: $g(t) = t^2$ is an A-Lipschitz function. These algebras turned out to be isomorphic to symmetrically normed Jordan ideals of C^*-algebras. If the completion $C(A)$ of A in the Ptak-Rajkov C^*-norm is not a CCR-algebra then all A-Lipschitz functions are Operator Lipschitzian. If it is a CCR-algebra then A-Lipschitz functions lie in the intersection of spaces of J-Lipschitz functions, where J are the symmetrically normed ideals of $B(H)$ associated with irreducible representations of $C(A)$.

[2000 MSC: 47A56,47L20]