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Noncommutative Uniform Algebras

A uniform algebra A is a Banach algebra such that $\left\|f^2\right\| = \|f\|^2$, for all $f \in A$. It is well known that any *complex* uniform Banach algebra is automatically commutative and is isometrically isomorphic with a subalgebra of $C_{\mathbb{C}}(X)$ [Hirschfeld and Żelazko, 1968]; such algebras are the most classical and well studied ones. Commutative, real uniform algebras have also been studied for years. Any such algebra A is isometrically isomorphic with a real subalgebra of $C_{\mathbb{C}}(X)$ for some compact set X; furthermore in most cases X can be just divided into three parts X_1, X_2, and X_3 such that $A_{|X_1}$ is a complex uniform algebra, $A_{|X_2}$ consists of complex conjugates of the functions from $A_{|X_1}$, and $A_{|X_3}$ is equal to $C_{\mathbb{R}}(X_3)$. In case of real uniform algebras the condition $\left\|f^2\right\| = \|f\|^2$ no longer implies commutativity - the algebra of quaternions serves as the simples counterexample. On the other hand there have been very little study of non commutative real uniform algebras. We show that any such algebra is isometrically isomorphic with a real subalgebra of $C_{\mathbb{H}}(X)$ - the algebra of all continuous functions defined on a compact set X and taking values in the field $\mathbb{H}$ of quaternions. We will also produce a general structural description of such algebras and address the question whether there are non trivial example of such algebras other then the direct sum of the entire algebra $C_{\mathbb{H}}(X)$ and a commutative real uniform algebra.

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