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Homomorphisms and composition operators on algebras of analytic functions

Let U and V be convex and balanced open subsets of the Banach spaces X and Y respectively. We study the following question: Given two Fréchet algebras of holomorphic functions of bounded type on U and V respectively that are algebra-isomorphic, can we deduce that X and Y (or X^* and Y^*) are isomorphic? We prove that if X^* or Y^* has the approximation property if the holomorphic functions that are weakly uniformly continuous, $H_{wu}(U)$ and $H_{wu}(V)$ are topologically algebra-isomorphic, then X^* and Y^* are isomorphic (the converse being true when U and V are the whole space). We get analogous results for $H_b(U)$ and $H_b(V)$, giving conditions under which an algebra-isomorphism between $H_b(X)$ and $H_b(Y)$ is equivalent to an isomorphism between X^* and Y^*. We also obtain characterizations of different algebra-homomorphisms as composition operators, study the structure of the spectrum of the algebras under consideration and show the existence of homomorphisms on $H_b(X)$ with pathological behaviors. We obtain applications of these results to spaces of germs.

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