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A Radon-Nikodym theorem for completely multi-positive linear maps and its applications

The concept of matricial order plays an important role to understand the infinite dimensional non-commutative structure of operator algebras. Completely positive maps, as the natural ordering attached to this structure have been studied extensively. In 1973, Paschke shows that a completely positive linear map from a *C**-algebra *A* to another *C**-algebra *B* induces a representation of the *C**-algebra *A* on a Hilbert *C**-module over *B* and proves a Radon-Nikodym type theorem which gives a description of the order relation in the set of all completely positive linear maps from a unital *C**-algebra *A* to a *W**-algebra *B*. Tsui extends the Paschke's results and he proves a Radon-Nikodym type theorem for completely positive linear maps between unital *C**-algebras. Using this theorem, he obtains characterizations of the extreme points in the set of all identity preserving completely positive linear maps from a *C**-algebra *A* to another *C**-algebra *B* and pure elements in the set of completely positive linear maps from *A* to *B* in terms of a self-dual Hilbert module structure induced by each completely positive linear map.
In 1999, Heo introduces the notion of completely multi-positive linear map between *C**-algebras (that is, an $n \times n$ matrix of bounded linear maps from a *C**-algebra *A* to another *C**-algebra *B* which verifies the complete positivity condition) and he shows that a completely multi-positive linear map from *A* to *B* induces a representation of *A* on a Hilbert *B* -module that generalizes the Paschke's construction. In this talk, we extend the Tsui's results for completely multi-positive linear maps from a locally *C**-algebra *A* to a *C**-algebra *B*, and finally, as an application of these results, we determine a certain class of extreme points in the set of all identity preserving completely positive linear maps from *A* to $M_n(B)$.

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