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Homologically trivial topological algebras

In this talk, we shall review some old and new results on structural and homological properties of certain classes of homologically trivial topological algebras. These include: contractible topological algebras, topological algebras of global dimension zero, and the class of biprojective topological algebras. As it is known, the simplest examples of contractible topological algebras are the full matrix algebras. The Cartesian product of an arbitrary family of full matrix algebras is contractible as well. There is a natural conjecture that an arbitrary contractible Arens-Michael algebra is topologically isomorphic to the Cartesian product of a certain family of full matrix algebras. This is established, at the moment, only under some additional assumptions (Helemskii, Selivanov, Fragoulopoulou). The class of biprojective topological algebras is much wider than the class of contractible algebras. Assuming certain conditions, such algebras can be represented as topological direct sums of topologically simple biprojective algebras. The latter are realized as tensor algebras generated by duality. The biprojectivity property of Fréchet algebras can be expressed in terms of their derivations: A is biprojective if and only if each continuous derivation of A with values in any Fréchet A-bimodule X is determined by a multiplier. We give explicit formulas for the corresponding expansions of derivations. It was first noted by Professor Helemskii that, if A is biprojective, then $\mathcal{H}(A,X) = 0$ for all A-bimodules. Now we can prove this fact in an explicit form. We have computed the cohomology groups of biprojective algebras with arbitrary coefficients. In particular, it is proved that the space $\mathcal{H}(A,X)$ is isomorphic to a quotient space of the space, $Q\mathcal{M}(X)$, of quasi-multipliers of X. Now we can also obtain this result by a method working with co cycles in the standard cohomology complex. As an application, in some concrete situations we can obtain a description of all singular extensions of A by X.

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