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Preserving parts of the spectra mappings between uniform algebras

Let $T : A \to B$ be a surjective mapping between two uniform algebras A and B. The search for conditions under which T is an isometric isomorphism has a long history. At various times Gleason, Kahane, Zelazko, Arens, Aupetit, Slodkowski, Kowalski, and Jarosz, among others, contributed to the problem. Recently Rao and Roy introduced the notion of multiplicatively spectrum-preserving mappings $T : A \to A$ of a uniform algebra A, based on the property $\sigma((Tf)(Tg)) = \sigma(fg),\ f, g \in A$, and have shown that multiplicatively spectrum-preserving mappings are necessarily isometric isomorphisms. Here we generalize and expand this result of Rao and Roy. Recall that for every $f \in A$ the *spectrum* $\sigma(f)$ of f is the image of $\mathcal{M}_A$ under f. Denote by $\sigma_b(f)$ the *set of values of f with maximum modulus*, i.e. $\sigma_b(f) = \{f(x) : |f(x)| = \|f\|,\ x \in \mathcal{M}_A\} = \sigma(f) \cap \{z \in \mathbb{C} : |z| = \|f\|\}$.

Theorem 1. *Let A and B be uniform algebras. If a surjective mapping $T : A \to B$ satisfies one of the following conditions*

(a): $T(1) = 1$, and $\sigma_b((Tf)\,(Tg)) = \sigma_b(fg)$, or,

(b): $T(0) = 0$, $\sigma_b(Tf + Tg) = \sigma_b(f + g)$ and $\sigma_b(|Tf| + |Tg|) = \sigma_b(|f| + |g|)$

for every $f, g \in A$, then T is an isometric algebra isomorphism between A and B.

In fact we show that under the above hypotheses the mapping T is the composition with a homeomorphism $\psi : Y \to X$, i.e. $Tf = f \circ \psi,\ f \in A$. In the proof we make use of the following additive version of Bishop's theorem for peaking functions: *If $E \subset X$ is a peaking set, $f \in A$, and $R = \|f\|$, then there exists an R-peaking function h for E so that $|f(x)| + |h(x)| < \max\limits_{y \in E} \big(|f(y)| + |h(y)|\big)$ for any $x \notin E$.*

[2000 MSC: 46J10]